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Constrained vs Unconstrained Labor Supply: The Economics of Dual Job Holding*

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Abstract

This paper develops a unified model of dual and unitary job holding based on a Stone-Geary utility function. The model incorporates both constrained and unconstrained labor supply. Panel data methods are adapted to accommodate multinomial selection into 6 mutually exclusive labor supply regimes. We derive and estimate the associated Slutsky equation wage and income elasticities using data from the British Household Panel Survey 1991-2008. Our study finds that the income and wage elasticities are much larger for labor supply to job 2 compared with job 1.

Keywords: dual job; labor supply; Stone-Geary

JEL classification codes: J22

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1 Introduction

Researchers have progressively extended labor supply theory in both static and dynamic dimensions to account for a richer variety of labor supply behavior. One fruitful area for research on labor supply is that of multiple job holding. One of most interesting aspects of multiple job holding is the motivation behind the decision to hold more than one job. A number of studies show that this decision is not only motivated by an hours constraint on the main job (also known as moonlighting), but also by a desire to hold a portfolio of jobs. Mostly, this literature has focused either on the determinants of each decision or on the labor supply for only one of the possible regimes. Considerably less attention has been paid to the development of a general labor supply model that allows for moonlighting as a response to an hours constraint on the main job and the joint determination of the hours supplied to two jobs when the decision to hold two jobs is not dictated by a constraint on the main job. In this paper we develop such a labor supply model based on a Stone-Geary utility function which allows us to model the choices of an individual who can hold up to two jobs.

Dual job holding is a pervasive phenomenon in many economies. Between 1994 and 2002, the weekly rate of dual job holding in U.K. was around 4.5 percent (ONS, 2002), but when computed on a monthly basis, the rate was found to be almost twice as high (Panos et al., 2011). Although Amuedo-Dorantes and Kimmel (2009) concludes that dual jobholding is pro-cyclical, the weekly rate of dual jobholding has remained stable around 5 percent in the U.S. during the 2000-2010 decade notwithstanding the recessions in 2001 and 2008 (Hipple, 2010). Dual job holding seems even more common in developing and transition economies, where the incidence of domestic production that takes place in the informal sector is typically higher than in developed countries. The rate of dual-jobholding Russian males doubled from the early to the mid 90's and stayed around 12 percent for the remainder of the decade (Foley, 1997). A survey of Tanzanian workers with a regular job in the formal economy found that more than half of them also held a job in the informal sector (Theisen, 2009).

Dual job holding is typically associated with an hour constraint on the main job. Firms often offer a fixed hours and wage employment package. If the number of hours that a firm offers falls short of the optimal number of hours that a utility maximizing worker would choose at the going wage, then a rational individual will take a second job under the condition that it pays more than his reservation wage on the second job. This is what we regard as the hours constraint motivation for holding multiple jobs, or "moonlighting". Moonlighting can be viewed as a substitute (perhaps temporary) for job search for a job with the optimal package of hours and wage rate. However, moonlighting alone cannot explain the behavior of all dual job holders. In fact, Allen (1998) concludes that unconstrained workers are more likely to have two jobs than are constrained workers. This result has led to a rich line of research on the motivation behind the decision to hold two jobs. Some individuals may decide to allocate their working time between two or more jobs because they have a personal preference for job differentiation.

For example, some workers may hold two jobs because jobs are heterogeneous and they are not perfect substitutes (Kimmel and Smith Conway, 2002). Others may hold a second job as a form of hedging against the risk of losing employment (Bell et al., 1997) or as a way to gradually transition to a new primary job, often self-employment (Panos et al., 2011). We group all reasons for holding two jobs that are not due to an hours constraint under the job portfolio hypothesis.

In this paper we expand on Kimmel and Smith Conway (2002) by using a Stone-Geary utility function to motivate the empirical work in the context of a dual job holding model.[1] The estimation is carried out for a sample of male workers from the British Household Panel Survey (BHPS). We use recent development in econometrics to model unobserved individual heterogeneity in a panel data setting. We derive the labor supply functions for unitary and dual job holders. For the latter, we distinguish whether they face a constraint on the hours offered on the main job or not. In addition we extend binary sample selection methods for panel data to multinomial selection. From our estimates, we compute the wage elasticities for each category of worker according to their constrained and dual-job status. While we confirm the results in the literature that the labor supply for unitary job holder is quite inelastic (Altonji and Paxson, 1988), the wage elasticities for dual job holders are quite large. This result has important policy implications as it suggests that the dual job holders' response to fiscal policy may be larger than for the entire population.

Section II reviews the literature on dual jobholding; Section III presents the theoretical framework used to derive our labor supplies equations. Section IV describes the data; Section V. discusses the estimation strategies of our empirical models; Section VI presents the empirical findings; and Section VII is a summary and conclusion.

2 Literature Review

Early theoretical work focused only on the hours constraint aspect of moonlighting (Perlman, 1966). Shishko and Rostker (1976) and Frederiksen et al. (2008) found that labor supply becomes more elastic to changes in the wage rate after accounting for the decision to moonlight. Extending the moonlighting model to a household labor supply framework, Krishnan (1990) found that the husband's decision to hold a second job is a substitute for the wife's decision to enter the labor market. Working under the same assumption that dual job holding is motivated solely by an hours constraint on the main job, Paxson and Sicherman (1998) concluded that moonlighting is a short-run solution to a situation of underemployment, while searching for a job that offers the target hours of work. However, the latter result is not supported by other studies that found that dual job holding is quite persistent over time and not just a short-run decision, thus casting doubt on the hours constraint hypothesis (Böheim and Taylor, 2004;

[1] Typically the Stone-Geary utility function is used to estimate expenditure functions for multiple commodity groups. See Chung (1994) for a review of the main studies based on a Stone-Geary utility function.

Panos et al., 2011).

A number of papers have tried to identify the determinants and hence the motives behind the decision to hold two jobs. Typically all studies conclude that while the probability of holding two jobs increases in the presence of hours or liquidity constraints (Abdukadir, 1992; Kimmel and Smith Conway, 2002; Panos et al., 2011), unconstrained workers are actually more likely to hold two jobs than constrained workers, thus suggesting that job portfolio motives may be even more important than the hours constraint (Allen, 1998; Böheim and Taylor, 2004). Exploiting the information contained in the 1991 Current Population Survey, Averett (2001) can identify the motives for holding two jobs. She classified as moonlighters all individuals who report working on a second job (1) to meet regular household expenses, (2) to pay off debts, (3) to save for the future or (4) to buy something special. She identified as dual job holders with job portfolio motives all individuals who report working on a second job (1) to get experience in a different occupation or to build a business, (2) to help out a friend or relative, (3) because he/she enjoys the work on the second job, and (4) other reasons. She estimated the probability of being a moonlighter, conditional on being a dual job holder, but she is unable to identify any specific determinant that is consistently significant across alternative models.

Only a handful of papers have actually attempted to estimate labor supply models that include the hours constraint and the job portfolio motive as alternative motives to working on a second job. Wu et al. (2009) includes an indicator for being satisfied with the hours worked on the main job in the second job hour equation, but failed to recognize that the specification of labor supply for moonlighters is different from that associated with the job portfolio hypothesis. In particular, the hours supplied on the first job should be included in the labor supply equation for the second job for moonlighters but not in the labor supply equation for the second job in the job portfolio model. To the best of our knowledge, Kimmel and Smith Conway (2002) is the only attempt that recognizes this important distinction. However, their data does not allow them to identify whether the decision to work on a second is motivated by an hours constraint. Consequently, they first estimate the probability that a moonlighter faces an hours constraint on the main occupation using a disequilibrium model. They then use the predicted probability of being a moonlighter to estimate the alternative labor supplies. Although they work with panel data, no attempt is made to control for individual unobserved heterogeneity.

3 Conceptual Framework

In this section we introduce the theoretical labor supply functions obtained from utility maximization as well as the key labor supply elasticities that arise from our model. Derivations underlying the elasticities are reported in the appendix.

Unconstrained dual job holder

Consider utility maximization for a multiple (dual) job holder who is not constrained in his choice of hours to work at two jobs. To make things concrete we will consider a Stone-Geary utility function for two different jobs or tasks in the absence of random disturbances:

$$U = (\gamma_1 - h_1^*)^{\alpha_1} \ (\gamma_2 \ - h_2^*)^{\alpha_2} \ (y^* - \gamma_3)^{1-\alpha_1-\alpha_2} \tag{1}$$

where $\alpha_1, \alpha_2, \gamma_1, \gamma_2, \gamma_3 > 0$, h_m^* represents the time allocated to job m, and y^* is income. The parameters γ_1 and γ_2 represent the upper bounds on the time that can be expended on jobs 1 and 2, and still have the utility function defined. They satisfy the following restriction:

$$\sum_{m=1}^{2} \gamma_m = T$$

where T is the total time available for work and leisure. The parameter γ_3 represents the lower bound on the amount of income necessary in order to have the utility function defined. The economic problem can be stated as

$$\begin{aligned}
\max_{h_1,h_2,y} U \ &= (\gamma_1 - h_1^*)^{\alpha_1} \ (\gamma_2 \ - h_2^*)^{\alpha_2} \ (y^* - \gamma_3)^{1-\alpha_1-\alpha_2} \\
\text{s.t. } y^* &= \sum_{m=1}^{2} w_m h_m^* + I, \\
0 < h_m^* \ &< \gamma_m, \ m = 1, 2 \text{ and} \\
\sum_{m=1}^{2} h_m^* &\leq T,
\end{aligned}$$

where w_m is the wage or pecuniary rewards to the mth job, and I is exogenous non-labor income.

The relationship between actual hours of labor supply, h_m, and planned hours, h_m^*, are given by $h_m^* = h_m - v_m$ or $h_m = h_m^* + v_m$, where v_m is a random error term. The properties of v_m will be considered under various scenarios described below.

It can be shown that the labor supply functions to the two jobs when $v_m = 0$ are given by

$$h_1^* = (1 - \alpha_1)\,\gamma_1 - \alpha_1\gamma_2\left(\frac{w_2}{w_1}\right) + \alpha_1\gamma_3\left(\frac{1}{w_1}\right) - \alpha_1\left(\frac{I}{w_1}\right) \tag{2}$$

$$h_2^* = (1 - \alpha_2)\,\gamma_2 - \alpha_2\gamma_1\left(\frac{w_1}{w_2}\right) + \alpha_2\gamma_3\left(\frac{1}{w_2}\right) - \alpha_2\left(\frac{I}{w_2}\right). \tag{3}$$

Accordingly, the earnings versions of the labor supply functions are expressed as

$$w_1 h_1^* = \alpha_1\gamma_3 + (1 - \alpha_1)\,\gamma_1 w_1 - \alpha_1\gamma_2 w_2 - \alpha_1 I \tag{4}$$

$$w_2 h_2^* = \alpha_2\gamma_3 + (1 - \alpha_2)\,\gamma_2 w_2 - \alpha_2\gamma_1 w_2 - \alpha_2 I. \tag{5}$$

The own wage Slutsky equation in elasticity form may be expressed by

$$\eta_{mm} = \eta^c_{mm} + \epsilon_{mm\mathrm{I}} \, ,$$

where

$$\begin{aligned}\eta_{mm} &= \frac{w_m}{h^*_m}\frac{\partial h^*_m}{\partial w_m} \\ &= \frac{\alpha_m}{w_m h^*_m}\left(\gamma_k w_k + \ I \ - \gamma_3\right) \gtreqless 0,\end{aligned}$$

is the uncompensated own wage elasticity for job m,

$$\begin{aligned}\eta^c_{mm} &= \frac{w_m}{h^*_m} S_{mm} \\ &= \frac{\alpha_m}{w_m h^*_m}\left(\gamma_k w_k + w_m h^*_m + \ I \ - \gamma_3\right) > 0,\end{aligned}$$

is the compensated own substitution effect elasticity for job m, S_{mm} is the compensated own substitution effect, and

$$\epsilon_{mm\mathrm{I}} \ = -\alpha_m \ < 0,$$

is the own wage income effect elasticity. We see that the effect of an uncompensated increase in the wage for job m can exhibit a positive, negative, or zero effect on labor supply to the mth job. An “inferior” job might be defined as one in which an increase in its wage leads to a reduction in labor supply to the given job and some combination of increases in leisure and labor supplied to the other job.

The pure income effect elasticity for job m is given by

$$\begin{aligned}\eta_{mI} \ &= \frac{I}{h_m}\frac{\partial h^*_m}{\partial I} \\ &= -\frac{\alpha_m I}{w_m h^*_m} < 0\end{aligned}$$

so that leisure is a normal good since an increase in non-labor income will reduce the labor supplied to both jobs and hence increase the consumption of leisure time.

The Slutsky equation for cross wage effects in elasticity form is given by

$$\eta_{mk} = \eta^c_{mk} + \epsilon_{mk\mathrm{I}},$$

where

$$\begin{aligned}\eta_{mk} &= \frac{w_k}{h_m^*}\frac{\partial h_m^*}{\partial w_k} \\ &= \frac{-\alpha_m \gamma_k w_k}{w_m h_m^*} < 0,\end{aligned}$$

is the uncompensated cross wage effect elasticity of labor supply to job m from a change in the wage for job k,

$$\begin{aligned}\eta_{mk}^c &= \frac{w_k}{h_m^*} S_{mk} \\ &= \frac{-\alpha_m w_k}{w_m h_m^*}\left(\gamma_k - h_k^*\right) < 0,\end{aligned}$$

is the compensated cross substitution effect elasticity, S_{mk} is the compensated cross substitution effect of a change in the wage on job k on labor supply to job m, and

$$\epsilon_{mkI} = \frac{-\alpha_m w_k h_k^*}{w_m h_m^*} < 0,$$

is cross-wage income effect elasticity. We see that both uncompensated and compensated increases in the wage for job k lead to reductions in labor supply to job m.

Unconstrained unitary job holders

For individuals who hold only one job, we condition on $h_2 = 0$ while assuming the same utility function as that of a dual job holder:

$$\begin{aligned}\max_{h_1, y} U \ &= (\gamma_1 - h_1^*)^{\alpha_1}\ (\gamma_2\ )^{\alpha_2}\ (y^* - \gamma_3)^{1-\alpha_1-\alpha_2} \\ \text{s.t. } y &= w_1 h_1^* + I, \\ 0 &< h_1^*\ < \gamma_1, \\ h_1^* &\leq T.\end{aligned}$$

Labor supply to job 1 in this case can be shown to be

$$h_1^* = \left(\frac{1-\alpha_1-\alpha_2}{1-\alpha_2}\right)\gamma_1 + \left(\frac{\alpha_1}{1-\alpha_2}\right)(\gamma_3)\left(\frac{1}{w_1}\right) - \left(\frac{\alpha_1}{1-\alpha_2}\right)\left(\frac{I}{w_1}\right) \tag{6}$$

or in terms of earnings

$$w_1 h_1^* = \left(\frac{1-\alpha_1-\alpha_2}{1-\alpha_2}\right)\gamma_1 w_1 + \left(\frac{\alpha_1}{1-\alpha_2}\right)(\gamma_3) - \left(\frac{\alpha_1}{1-\alpha_2}\right) I. \tag{7}$$

The own wage Slutsky equation in elasticity form for job 1 when not working a second job is expressed as

$$\eta_{11}\left|_{h_2=0}\right. = \eta_{11}^c\left|_{h_2=0}\right. + \epsilon_{11I}\left|_{h_2=0}\right. ,$$

where

$$\begin{aligned}
\eta_{11}\left|_{h_2=0}\right. &= \frac{w_1}{h_1^*}\frac{\partial h_1^*}{\partial w_1}\left|_{h_2=0}\right. \\
&= \left(\frac{\alpha_1}{1-\alpha_2}\right)\left(\frac{1}{w_1 h_1^*}\right)\left(I \ -\gamma_3\right) \gtreqless 0,
\end{aligned}$$

is the uncompensated own wage effect elasticity for job 1,

$$\begin{aligned}
\eta_{11}^c\left|_{h_2=0}\right. &= \frac{w_1}{h_1^*}S_{11}\left|_{h_2=0}\right. \\
&= \left(\frac{\alpha_1}{1-\alpha_2}\right)\left(\frac{1}{w_1 h_1}\right)(w_1 h_1 + I - \gamma_3) > 0,
\end{aligned}$$

is the compensated own substitution effect elasticity for job 1, $S_{11}\left|_{h_2=0}\right.$ is the own compensated substitution effect, and

$$\epsilon_{11I}\left|_{h_2=0}\right. = \frac{-\alpha_1}{1-\alpha_2} < 0,$$

is the income effect elasticity from the own wage.

The pure income effect elasticity for h_1^* is determined by

$$\begin{aligned}
\eta_{1I}\left|_{h_2=0}\right. &= \frac{I}{h_1^*}\frac{\partial h_1^*}{\partial I}\left|_{h_2=0}\right. \\
&= \left(\frac{-\alpha_1}{1-\alpha_2}\right)\left(\frac{I}{w_1 h_1^*}\right) < 0.
\end{aligned}$$

<u>Constrained dual job holder</u>

We will assume that constraints on labor supply for dual job holders apply to job 1, i.e. people are constrained either because they desire more hours on job 1 (under-employed) or they desire fewer hours on job 1 (over-employed). It is therefore assumed that they are working their desired hours on job 2 given their constrained hours in job 1. For an individual who is constrained at $h_1 = \dot{h}_1$, the utility maximization problem becomes

$$\begin{aligned}
&\max_{h_2, y} U = \left(\gamma_1 - \dot{h}_1\right)^{\alpha_1} \left(\gamma_2 \ - h_2^*\right)^{\alpha_2} \left(y^* - \gamma_3\right)^{1-\alpha_1-\alpha_2} \\
&\quad \text{s.t. } y = w_2 h_2^* + w_1 \dot{h}_1 + I, \\
&\qquad 0 \le h_2^* \ < \gamma_2, \ \ 0 \le \dot{h}_1 \ < \gamma_1, \text{ and} \\
&\dot{h}_1 + h_2^* \le T,
\end{aligned}$$

While labor supply to job 1 is fixed at $\dot{h}_1$, expected labor supply to job 2 is determined according to

$$h_2^* = \left(\frac{1-\alpha_1-\alpha_2}{1-\alpha_1}\right)\gamma_2 + \left(\frac{\alpha_2\gamma_3}{1-\alpha_1}\right)\left(\frac{1}{w_2}\right) - \left(\frac{\alpha_2}{1-\alpha_1}\right)\left(\frac{w_1\dot{h}_1+I}{w_2}\right). \tag{8}$$

In terms of expected earnings, labor supply to job 2 would simply be

$$w_2 h_2^* = \left(\frac{1-\alpha_1-\alpha_2}{1-\alpha_1}\right)\gamma_2 w_2 + \left(\frac{\alpha_2\gamma_3}{1-\alpha_1}\right) - \left(\frac{\alpha_2}{1-\alpha_1}\right)\left(w_1\dot{h}_1+I\right). \tag{9}$$

The own wage Slutsky equation in elasticity form for job 2 when constrained on job 1 may be expressed as

$$\eta_{22}\left|_{h_1=\dot{h}_1}\right. = \eta_{22}^c\left|_{h_1=\dot{h}_1}\right. + \epsilon_{22I}\left|_{h_1=\dot{h}_1}\right. ,$$

where

$$\begin{aligned}\eta_{22}\left|_{h_1=\dot{h}_1}\right. &= \frac{w_2}{h_2^*}\frac{\partial h_2^*}{\partial w_2}\left|_{h_1=\dot{h}_1}\right. \\ &= \left(\frac{\alpha_2}{1-\alpha_1}\right)\left(\frac{1}{w_2 h_2^*}\right)\left(w_1\dot{h}_1 + \ I \ - \gamma_3\right) \gtreqless 0,\end{aligned}$$

is the uncompensated wage uncompensated own wage elasticity for job 2,

$$\begin{aligned}\eta_{22}^c\left|_{h_1=\dot{h}_1}\right. &= \frac{w_2}{h_2^*}S_{22}\left|_{h_1=\dot{h}_1}\right. \\ &= \left(\frac{\alpha_2}{1-\alpha_1}\right)\left(\frac{1}{w_2 h_2^*}\right)\left(w_1\dot{h}_1 + w_2 h_2^* + I - \gamma_3\right) > 0,\end{aligned}$$

is the compensated own substitution elasticity, $S_{22}\left|_{h_1=\dot{h}_1}\right.$ is the compensated own substitution effect, and

$$\begin{aligned}\epsilon_{22I}\left|_{h_1=\dot{h}_1}\right. &= \eta_{22}\left|_{h_1=\dot{h}_1}\right. - \eta_{22}^c\left|_{h_1=\dot{h}_1}\right. \\ &= \frac{-\alpha_2}{1-\alpha_1} < 0,\end{aligned}$$

is the income effect elasticity from the own wage.

The pure income effect elasticity for h_2^* is determined by

$$\begin{aligned}\eta_{2I}\left|_{h_1=\dot{h}_1}\right. &= \frac{I}{h_2}\frac{\partial h_2^*}{\partial I}\left|_{h_1=\dot{h}_1}\right. \\ &= \left(\frac{-\alpha_2}{1-\alpha_1}\right)\left(\frac{I}{w_2 h_2^*}\right) < 0.\end{aligned}$$

It can be shown that the compensated cross-substitution effect of wages on job 1 on labor supply to job 2 is necessarily zero when hours are constrained in job 1. This means that wages on job 1 can only have income effects. Hence the uncompensated cross-wage elasticity of w_1 on h_2 is the same as the cross-wage income effect elasticity:

$$\eta_{21}\left|_{h_1=\dot{h}_1}\right. = \frac{w_1}{h_2}\frac{\partial h_2^*}{\partial w_1}|_{h_1=\dot{h}_1} = \epsilon_{21I}\left|_{h_1=\dot{h}_1}\right.$$
$$= \left(\frac{-\alpha_2}{1-\alpha_1}\right)\left(\frac{w_1\dot{h}_1}{w_2 h_2^*}\right) < 0.$$

Easing of the hours constraint on job 1 would decrease the hours supplied to job 1 for an over-employed worker and increase the hours supplied to job 1 for an under-employed worker. The Stone-Geary dual labor supply model predicts that the unconstrained labor supply to job 2 is affected by the hours constraints on job 1. Inspection of equation (8) reveals that

$$\frac{\partial h_2^*}{\partial \dot{h}_1} = -\left(\frac{\alpha_2}{1-\alpha_1}\right)\left(\frac{w_1}{w_2}\right) < 0. \tag{10}$$

For an over-employed worker, (10) implies that as $\dot{h}_1 \downarrow h_1^*$, $h_2 \uparrow h_2^*$. Therefore, over-employed workers would increase their labor supply to job 2 if the excess hours constraint on job 1 were relaxed. In other words, these workers originally reduce their hours in job 2 to compensate for over-employment in job 1. By the same token, (10) implies that as $\dot{h}_1 \uparrow h_1^*$, $h_2 \downarrow h_2^*$. Therefore, under-employed workers would decrease their hours in job 2 in response to an easing of the hours constraint in job 1 because originally they increased labor supply to job 2 in order to compensate for underemployment in job 1.

Constrained unitary job holder

For a constrained unitary job holder, the hours worked $(\dot{h}_1)$ are treated as exogenous. We view the constrained hours as a temporary disequilibrium from desired hours over either one or two jobs. As in the case of hours constraints on job 1 for a dual job holder, easing of the hours constraint on job 1 for a unitary job holder would decrease the hours supplied to job 1 for an over-employed worker and increase the hours supplied to job 1 for an under-employed worker.

4 Data

The estimation of our model is carried out using data from the British Household Panel Survey (BHPS). The survey was started in 1991 with a sample of some 5,500 household randomly drawn from all areas of Great Britain. To this initial sample, an over-sample of 1,500 households form Scotland and Wales was added in 1999 and a sample of 2,000 households from Northern Ireland was added in 2001. Individuals are followed over time through an annual questionnaire. The survey asked whether in the month preceding the interview the respondent had worked on a second job. Figure 1 shows the incidence of dual job holding year by year from 1991 to 2008. The sample is restricted to prime age working men (age 18 to 65) who are not enrolled in school, to avoid standard selection problems associated with the labor supply decision of women or individuals eligible for retirement. We also exclude self-employed individuals on the main job. For the purposes of the BHPS, the main job for a dual job holder is the job in which one works the most hours. Dual job holding picked up in 1996 and slowly declined afterward with a sharp drop in 2001 in correspondence to the expansion of the survey to a sample of Irish households.

Importantly for the scope of this study, BHPS contains information about the presence of an hours constraint on the main job. Specifically respondents were asked whether they would have liked to work more, less, or the same hours assuming that they would be paid the same amount per hour. Since this question was asked directly after respondents reported their hours of work on the main job, we interpret the answer to this question as an indicator for an hours constraint on the main job. Accordingly for each type of job holder (unitary or dual) we can identify if he is constrained on the main job. In the end we have 6 possible cases with the number of observations for each category: 1) unitary job holders who work their desired amount of hours on the main job - unconstrained single job holders; 2) unitary job holders who would have liked to work more hours on the main job (under-employed); 3) unitary job holders who would have liked to work less hours on the main job (over-employed); 4) dual job holders who work their desired hours on the main job - unconstrained dual job holders; 5) dual job holders who are under-employed on their first job; 6) dual job holders who are over-employed on their main job.

Table 1 reports means for variables in our analyses. While most of the variables are straightforward, some may require explanation about how they were constructed. The wage rate was calculated by dividing the monthly earnings by the usual hours worked on the relevant job times four. This procedure was preferred to the self reported information on the hourly wage rate because it guarantees internal consistency between the estimation of the hours and the earnings equations. To mitigate the effect of outliers, we deleted from our sample individuals who earn less than £1/hour or more than £100/hr. Moreover we drop individuals with reported hours of work outside the 1 to 99 percentile of the distribution of hours worked in the sample. Non-labor income is the sum of all state benefits (including pension), money transfer, and income from rent, saving, and investment.

After excluding observations with missing data for any variables in the models, we are left with about 7,970 individuals with an average of 5.6 observations per individual for a total of 44,572 observation points. We have complete information on dual job holders in 2,556 cases, which account for about 5.7% of all the observations in our estimation sample. Almost 60% of dual job holding episodes are associated with no hour constraints on the main job; another 32% of the dual job holding episodes is associated with workers who are over-employed on the main job, and the remaining 8% of dual job holding episodes is associated with workers who are under-employed on the main job. This first result indicates that the usual explanation for holding two jobs, i.e. the need to fulfill an unmet hours target on the main job, does not seem to fit well with the stylized facts in the UK. First only 8% of our dual job holding observations can be classified under the under-employed hours constrained model. Second, the under-employed hours constrained model cannot explain why so many dual job holders actually wished they were working less hours on the main job: if a worker is over-employed on the main job, why would he take a second job? The job portfolio model offers a reasonable explanation for this finding. The decision to hold two jobs is independent from the hours worked on the main job. Jobs are heterogeneous for a variety of reason and that is why some workers prefer to allocate their target hours of work over a portfolio of jobs. Long contractual hours on the main job may actually bring a worker above the desired target of hours. Although the portfolio model seems to better serve the stylized facts in the UK, one cannot disregard that only 6 percent of the sample of single job episodes consider themselves under-employed. This is somewhat lower than the incidence of under-employment among dual job holders, thus suggesting that individuals do get a second job in response to a situation of under-employment on the main job.

Not surprisingly, we found that under-employed individuals work less hours on the main job than unconstrained workers while over-employed individuals work more hours on the main job than unconstrained workers. On average, dual job holders work less hours on their main job, but after adding the hours supplied on the second job, dual job holders work more total hours per week. The average hourly wage on the main job for unitary job holders is higher than for dual job holders. For dual job holders, the hourly wage rate and monthly earnings on the second job are typically lower than on job 1. The portfolio hypothesis does not impose any restriction on the relationship between the rates of pay between the two jobs, as such the findings in Table 1 are always consistent with that hypothesis. The insufficient-hours constraint model predicts that the rate of pay on the main job should be higher than the rate of pay on the second job. Hence, both the hourly wage and the monthly earnings observed are also consistent with the insufficient-hours constrained model. Compared with their unconstrained fellow workers, the average hourly wage of under-employed workers is lower and the average hourly wage of over-employed is higher.

Unitary and dual job holders differ on a number of socio-economic dimensions. For example, dual job holders tend to be younger that unitary job holders and less likely to be married. Moreover, under-employed workers (both unitary and dual job holders) seems to be less educated than the other two classes of workers: only 19% of under-employed unitary job holders and 17% of under-employed dual job holders have some degree above A level. The same rates for over-employed workers are 29% and 32% respectively. Hence, it could well be that some underlying selection process determines whether a workers falls into one of the six categories in a systematic way. We address this issue in the following section.

5 Empirical Model

Our sample is partitioned according to six mutually exclusive outcomes: (1) unconstrained dual job holders, (2) unconstrained unitary job holders, (3) constrained dual job holders desiring fewer hours on job 1, (4) constrained dual job holders desiring more hours on job 1, (5) constrained unitary job holders desiring fewer hours , and (6) constrained unitary job holders desiring more hours. Because constrained hours on job 1 are treated as exogenous, we do not estimate corresponding labor supply functions in these cases. This leaves us with five labor supply functions to estimate that span four selection regimes: h_1 and h_2 for case (1), h_1 for case (2), h_2 for case (3), and h_2 for case (4). Hours are measured as hours per week, wages are measured as hourly wage rates, and non-labor and total income are measured on a weekly basis. All monetary variables are expressed in terms of 2008 prices.

Our analysis extends the sample selection approaches of Lee (1983), Wooldridge(1995; 2010), and Dustmann and Rochina-Barrachina (2007) to multivariate selection in a panel data setting. Let s_{it} represent a variable that assumes the values $0, 1, ..., 5$ corresponding to the six job holding regimes. We can equivalently define indicator variables corresponding to these six labor supply regimes: $s_{itj} = 1\left[s_{it} = j\right]$. Following Wooldridge (2010, pp.653-654), we specify that $P\left(s_{it} = j \mid x_{it}^r, c_i^r\right) = P\left(s_{it} = j \mid \mathbf{x}_i^r, c_i^r\right)$, $j = 0, 1, ..., 5$ is determined according to a multinomial logit model with unobserved individual effects, where $r = h, e$ for hours or earnings, x_{it}^r is a vector of all exogenous variables in the model for which there are observations $\forall\ i$ and t, $\mathbf{x}_i^r$ is the vector of all observations for x_{it}^r for the ith individual, and c_i^r is unobserved heterogeneity. For the earnings model $(r = e)$, the observed variables are the constant term, w_1, I, age, educ (a vector of educational dummy variables), marital status (MS=1 if married), and number of dependent children (DP). In the case of the hours model $(r = h)$, w_1 and I are replaced by $\dfrac{1}{w_1}$ and $\dfrac{I}{w_1}$.

The first stage of our panel data estimation of the dual labor supply model is pooled multinomial logit. From these results we construct the appropriate Inverse Mill Ratio (IMR) variables that are to be added as regressors in the five labor supply equations. Because of complications introduced by the presence of unobserved heterogeneity c_i^r, we follow Wooldridge in assuming that the conditional distributions of $c_i^r \mid \mathbf{x}_i^r$ and $c_i^r \mid \bar{x}_i^r$ are the same, where $\bar{x}_i^r$ is a vector of the time averages of the variables in $\mathbf{x}_i^r$. This equality of conditional distributions implies

$$P\left(s_{it} = j \mid \mathbf{x}_i^r\right) = P\left(s_{it} = j \mid x_{it}^r, \bar{x}_i^r\right)\ \forall i \text{ and } t.$$

The assumed multinomial selection model generates probabilities according to

$$\begin{aligned} P_{ijt}^r &= P\left(s_{it} = j \mid x_{it}^r, \bar{x}_i^r\right),\ j = 1, ..., 5 \\ &= \Lambda\left(x_{it}^r, \bar{x}_i^r, \beta_j^r\right) \\ P_{i0t}^r &= 1 - \textstyle\sum_{j=1}^5 P_{ijt}^r, \end{aligned}$$

where β_j^r is the multinomial logit parameter vector for outcome j. Let $z_{ijt}^r = \Phi^{-1}\left(P_{ijt}^r\right)$, where Φ^{-1} is the inverse standard normal CDF. It is clear that $\Phi\left(z_{ijt}^r\right) = P_{ijt}^r = \Lambda\left(x_{it}^r, \bar{x}_i^r, \beta_j^r\right)$. The corresponding Inverse Mills Ratios are calculated as $\lambda_{ijt}^r = \dfrac{\phi\left(z_{ijt}^r\right)}{\Phi\left(z_{ijt}^r\right)}$.

Let v^r_{mlit} represent the sum of an unobserved individual effect for labor supply and an idiosyncratic error term, where $m = 1, 2$ for job 1 or job 2, $l = 1, 2, 3, 4$ indexes the four labor supply selection regimes, $i = 1, ..., n$, and $t = 1, ..., T_i$. The error structure for each labor supply regime can be characterized by (see Wooldridge 2010, pp.832-837)

$$E\left(v^r_{mlit} | q^r_{mlit}, \bar{x}^r_i, l_{it}\right) = \theta^r_{ml} \lambda^r_{mlit} + \bar{Z}^r_{mli} \pi_{ml},$$

where q^r_{mlit} is the restricted variable in the labor supply function that arises from utility maximization, $\bar{Z}^r_{mli}$ is a vector of time averaged means, and π_{ml} is a conforming parameter vector. If we let $u^r_{mlit} = v^r_{mli} - E\left(v^r_{mlit} | q^r_{mlit}, \bar{x}^r_i, l_{it}\right)$, then the error process for each labor supply equation is specified by

$$v^r_{mlit} = \theta^r_{ml} \lambda^r_{mlit} + \bar{Z}^r_{mli} \pi_{ml} + u^r_{mlit}.$$

Depending upon the assumed data generating process, labor supply can be represented by either hours or earnings. The resulting labor supply equations are jointly estimated by pooled, non-linear SUR with cross-equation restrictions on the parameters α_1 and α_2. In practice the Inverse Mills Ratios (IMR's) are replaced by their estimated values $\hat{\lambda}^r_{mlit}$ obtained from the multinomial logit model. Estimated standard errors are bootstrapped.

We estimate the Stone-Geary model's boundary parameters $\gamma_1, \gamma_2,$ and γ_3 directly from our panel data sample. Let $\tilde{\gamma}_1$ be the highest integer value that satisfies $h^{\max}_1 < \tilde{\gamma}_1 \leq 1 + h^{\max}_1$ for the combined samples for all workers who work job 1 over all periods; let $\tilde{\gamma}_2$ be the highest integer value that satisfies $h^{\max}_2 < \tilde{\gamma}_2 \leq 1 + h^{\max}_2$ for the combined samples for all workers who work job 2 over all periods; and let $\tilde{\gamma}_3$ be the lowest integer value that satisfies $y^{\min} - 1 \leq \tilde{\gamma}_3 < y^{\min}$ for the combined samples for all workers over all periods, where $h^{\max}_m$ is the maximum observed hours of work for job m and $y^{\min}$ is the lowest observed income.

Although the estimated empirical model corresponding to the hours specification yielded theoretically nonsensical results, the estimated earnings specification of our labor supply model produced quite reasonable estimates of the utility function parameters. Therefore, we assume that the true data generating process is best captured by the empirical earnings model. Our specification of the empirical earnings model is presented below. For the interested reader, the empirical specification of the hours model is presented in the technical appendix to the paper.

Empirical Model for Earnings

In the case of the multinomial logit model with the earnings specification, we define

$$x_{it} = (w_{1it}, I_{it}, \text{Age}_{it}, \text{Educ}_{it}, \text{MS}_{it}, \text{DP}_{it})$$
$$\bar{x}_i = \left(1, \bar{w}_{1i}, \bar{I}_i, \overline{\text{Age}}_i, \overline{\text{Educ}}_i, \overline{\text{MS}}_i, \overline{\text{DP}}_i\right).$$

The labor supply functions for the earnings model are specified below.

Unconstrained dual job holders

$$w_{1it}\left(h_{1it}-\tilde{\gamma}_1\right)=\alpha_1 q^e_{1it}+\theta^e_{11}\hat{\lambda}^e_{1it}+\bar{Z}^e_{1i}\pi^e_{11}+\ u^e_{11it} \tag{11}$$

$$w_{2it}\left(h_{2it}-\tilde{\gamma}_2\right)=\alpha_2 q^e_{1it}+\theta^e_{21}\hat{\lambda}^e_{1it}+\bar{Z}^e_{1i}\pi^e_{21}+\ u^e_{21it}. \tag{12}$$

where

$$\begin{aligned} q^e_{11it}&=q^e_{21it}=q^e_{1it}\\ &=\tilde{\gamma}_3-\tilde{\gamma}_1 w_{1it}-\tilde{\gamma}_2 w_{2it}-I_{it}, \end{aligned}$$

$$\begin{aligned} \bar{Z}^e_{11i}&=\bar{Z}^e_{21i}=\bar{Z}^e_{1i}\\ &=\left(1,\bar{w}_{1i},\bar{w}_{2i},\bar{I}_i,\overline{\text{Age}}_i,\overline{\text{Educ}}_i,\overline{\text{MS}}_i,\overline{\text{DP}}_i\right), \end{aligned}$$

and π_{11} and π_{21} are the corresponding parameter vectors.

Unconstrained single job holders

$$w_{1it}\left(h_{1it}-\tilde{\gamma}_1\right)=\left(\frac{\alpha_1}{1-\alpha_2}\right)q^e_{12it}+\theta^e_{12}\hat{\lambda}^e_{2it}+\bar{Z}^e_{12i}\pi^e_{12}+\ u^e_{12it} \tag{13}$$

where

$$q^e_{12it}=\tilde{\gamma}_3-\tilde{\gamma}_1 w_{1it}-I_{it},$$

$$\bar{Z}^e_{12i}=\left(1,\bar{w}_{1i},\bar{I}_i,\overline{\text{Age}}_i,\overline{\text{Educ}}_i,\overline{\text{MS}}_i,\overline{\text{DP}}_i\right),$$

and π^e_{12} is the corresponding parameter vector.

Constrained dual job holders

Constrained dual job holders desiring either fewer or more hours:

$$w_{2it}\left(h_{2it}-\tilde{\gamma}_2\right)=\left(\frac{\alpha_2}{1-\alpha_1}\right)q^e_{2it}\ +\theta^e_{23}\hat{\lambda}^e_{23it}+\bar{Z}^e_{3i}\pi^e_{23}+u^e_{23it}\ \text{(over-employed)} \tag{14}$$

$$w_{2it}\left(h_{2i}-\tilde{\gamma}_2\right)=\left(\frac{\alpha_2}{1-\alpha_1}\right)q^e_{2it}\ +\theta^e_{24}\hat{\lambda}^e_{24it}+\bar{Z}^e_{3i}\pi^e_{24}+u^e_{24it}\ \text{(under-employed).} \tag{15}$$

where $\dot{h}_{1it}$ is the constrained hours on job 1,

$$\begin{aligned} q^e_{23it}&=q^e_{24it}=q^e_{2it}\\ &=\tilde{\gamma}_3-\tilde{\gamma}_2 w_{2it}-\left(w_{1it}\dot{h}_{1it}+I_{it}\right), \end{aligned}$$

$$\bar{Z}^e_{23i} = \bar{Z}^e_{24i} = \bar{Z}^e_{3i} = \left(1, \bar{w}_{2i}, \overline{w_{1i}\dot{h}_{1i}}, \bar{I}_i, \overline{\text{Age}}_i, \overline{\text{Educ}}_i, \overline{\text{MS}}_i, \overline{\text{DP}}_i\right),$$

and π^e_{23} and π^e_{24} are the corresponding parameter vectors.

6 Empirical Results

Table 2 reports the estimated multinomial logit model of labor supply selection for panel data. The estimated effects are relative to being an unconstrained unitary job holder. Higher job 1 wages and age are associated with reduced odds that a worker will be unconstrained as a dual job holder or underemployed but raise the odds of being an over-employed unitary job holder relative to being an unconstrained unitary job holder. Number of children and education do not appear to have systematic effects (cet. par.) on selection into labor supply regime except that the odds of being an over-employed dual job holder relative to an unconstrained unitary job holder increase with number of children and decrease with the lowest educational level. Being married is associated with increased odds of being an over-employed unitary job holder while non-labor income increases the odds of being an under-employed unitary job holder relative to being an unconstrained unitary job holder.

Estimates of the basic parameters of the labor supply model are reported in Table 3. The boundary parameters obtained from the sample add to 166 hours per week which is nearly identical to the 168 hour physical limit. The estimated values of α_1 and α_2 satisfy theoretical restrictions, i.e. they are positive and bounded on the unit interval. Furthermore, $\hat{\alpha}_2 > \hat{\alpha}_1$ implies that for a dual job holder utility is more responsive to changes in time not spent working on job 2 than to changes in time not spent working on job 1. Individuals who are selected into working in job 1 as either an unconstrained dual job holder or as an unconstrained unitary job holder are types who would work fewer hours in job 1. On the other hand workers who work two jobs are types who would work more hours on job 2, whether constrained or unconstrained.

In Table 4 we report the estimated labor supply elasticities evaluated at the sample mean values of the variables for each labor supply regime[2]. Theoretical restrictions on the labor supply elasticities are satisfied in every case. There is no theoretical prediction for uncompensated own wage elasticities, but these turn out to be positive without exception. Because the substitution effects dominate the income effects, there is no incidence of backward bending supply curves at the mean. In the case of the unconstrained unitary job holders, the income effect largely offsets the substitution effect so that the uncompensated labor supply elasticity is quite small. Among unconstrained dual job holders, the wage elasticities of labor

[2] We also estimated the labor supply elasticities at the sample median values of the variables as well as the average and the median of the individual estimated elasticities. With one minor exception, the results are qualitatively the same. However, the average of the individual elasticities produced relatively large magnitudes for labor supply elasticities on the second job for constrained dual job holders. See table A.1-A.3 in the appendix for more details.

supply to the second job are much larger than those associated with the main job. As can be seen from the wage elasticity formulas for this case, this finding stems from the fact that $\hat{\alpha}_2 > \hat{\alpha}_1$, $\bar{w}_1 > \bar{w}_2$, and $\hat{h}_1 > \hat{h}_2$.[3] It is also the case that the pure income effect and wage income effect elasticities are larger in absolute value on the second job. Among constrained dual job holders, the wage elasticities are slightly smaller for the under-employed as compared with the over-employed.

7 Summary and Conclusion

Using a Stone-Geary utility function we derive a more general model of labor supply that allows for workers to take on a second job. Our model is general in the sense that the reason for holding two jobs is not restricted to an hours constraint on the main job. We adopt the weekly earning version of our model because it yields the most plausible results. For the estimation we use data from the BHPS, a unique dataset that contains not only information about the second job, but also information about the hours constraint on the main job. We take advantage of the panel nature of this dataset and seek to model unobserved heterogeneity by extending Wooldridge (2010) to a multinomial logit selection equation.

From the results of our earnings equations, we compute the labor supply elasticities. We found that labor supply to job 2 is more responsive to changes in own and cross wages than labor supply to job 1. This is true not only in percentage terms (elasticities) but also in levels (number of hours). In fact, while job 1 is consistently found to be inelastic, the elasticities on job 2 are always well above 1 (see Table 4). The change in levels is a function of both the elasticities and the average hours worked on each job. While individuals on average work longer hours on job 1, the differences between the elasticities on the two jobs are so large that the change in the hours worked on the job 2 is always greater than the change in the hours worked on job 1. Similarly, we find that a rise in non-labor income would have little effect on the hours supplied to job 1, but it would reduce the hours supplied to job 2. This effect is largest (in absolute value) for underemployed workers, most likely because an increase in non-labor income would lead to a reduction in the desired hours of work and as such narrow the gap between the desired and offered hours of work. This finding supports the argument that job 2 is the marginal job and, as such, the hours supplied to job 2 should be more responsive to changes.

This study has potentially important implications for fiscal policies. In Table 4, we found that the uncompensated cross-wage elasticity almost completely offsets the own-wage elasticity. This result seems to suggest that a proportionate change in the wage rates on both jobs would leave total labor supply unchanged, lending some support to the usual argument that changes in the income tax system have no effect on labor supply, at least as far as the effects of marginal tax rates on wages for dual job holders are concerned. However, this may not always be the case. In fact a disproportioned percentage of dual job

[3] Rather than use the sample mean hours for our elasticity calculations, we use the predicted hours from the estimated labor supply models evaluated at the sample mean values of the RHS variables. Because our estimation strategy does not force the labor supply equations to pass through the sample mean, our approach restricts the estimated elasticities to be evaluated somewhere along the estimated labor supply curves.

holders are self-employed on the second job (12.6% on job 1 versus 45.3% on job 2). Given that often the tax treatment of self-employed income is different than the tax treatment for wages and salaries, these differences in the income tax system could ultimately affect the number of hours worked on the second job.

A natural extension of this work would be the estimation of a similar labor supply model for women. One might also consider developing a dynamic model of dual labor supply to fully exploit the panel nature of the dataset.

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Table 1: Summary Statistics (mean)

	Single job holders			Dual job holders		
Variable	**Unconstrained**	**Underemployed**	**Overemployed**	**Unconstrained**	**Underemployed**	**Overemployed**
Weekly hours worked on job 1	38.74	36.78	40.62	37.88	35.39	40.14
Weekly hours worked on job 2				24.81	25.39	24.16
Weekly non-labor income	68.68	70.34	66.47	72.86	68.72	69.14
Weekly earnings on job 1	430.17	325.48	497.05	390.59	290.18	465.33
Wage rate on job 1	12.86	10.30	14.36	12.05	9.61	13.74
Weekly earnings on job 2				80.00	71.24	98.72
Wage rate on job 2				5.67	4.13	7.18
Age	37.95	33.15	40.86	36.76	32.30	38.64
Educational attainment						
Higher degree (omitted)	0.04	0.02	0.04	0.05	0.01	0.06
1st degree	0.14	0.10	0.15	0.13	0.12	0.17
HND, HNC, teaching	0.09	0.07	0.10	0.08	0.04	0.09
A level	0.24	0.26	0.23	0.23	0.33	0.23
O level	0.26	0.28	0.24	0.29	0.32	0.26
CSE	0.07	0.08	0.06	0.07	0.06	0.06
None of these	0.17	0.18	0.19	0.14	0.11	0.13
Married (=1)	0.72	0.60	0.81	0.70	0.59	0.78
Number of children	0.67	0.72	0.70	0.76	0.65	0.84
Number of individuals	4532	572	2362	309	45	150
Number of observations	24634	2542	14840	1514	211	831

Notes: Based on British Household Panel Survey (1991-2008). All income variables are expressed in 2008 prices.

Table 2: Multinomial Logit Regression

	Single job holders			Dual job holders		
	Unconstrained	**Underemployed**	**Overemployed**	**Unconstrained**	**Underemployed**	**Overemployed**
Wage rate (job 1)	–	-0.048∗	0.006†	-0.026†	-0.161∗	-0.015
		(0.011)	(0.003)	(0.011)	(0.024)	(0.010)
Age	–	-0.068∗	0.009∗	-0.017‡	-0.059∗	-0.021‡
		(0.008)	(0.003)	(0.009)	(0.023)	(0.011)
N of children	–	-0.013	-0.019	0.008	0.188	0.117‡
		(0.042)	(0.021)	(0.051)	(0.119)	(0.068)
1st degree	–	-0.468	0.229	-1.377†	1.277	-0.174
		(0.486)	(0.216)	(0.554)	(1.213)	(0.298)
HND, HNC, teaching	–	-0.139	0.131	-0.823	-0.510	1.009
		(0.585)	(0.297)	(0.736)	(1.390)	(0.710)
A level	–	-0.463	0.019	-0.343	0.586	-0.467
		(0.515)	(0.250)	(0.610)	(1.052)	(0.474)
O level	–	-0.490	0.164	-0.220	0.635	-0.538
		(0.554)	(0.273)	(0.688)	(1.413)	(0.665)
CSE	–	-0.701	0.244	-1.989‡	-1.733	-1.982‡
		(0.768)	(0.407)	(1.124)	(1.732)	(1.138)
None of these	–	-0.938	-0.399	-0.553	-1.991	-0.346
		(0.660)	(0.332)	(0.754)	(1.431)	(1.071)
Married (=1)	–	0.078	0.231∗	-0.129	-0.356	-0.008
		(0.100)	(0.054)	(0.131)	(0.342)	(0.182)
Weekly non-labor income/100	–	0.041‡	-0.001	0.024	-0.090	-0.064
		(0.025)	(0.014)	(0.030)	(0.083)	(0.058)
Constant	–	-0.882∗	-1.944∗	-1.997∗	-3.697∗	-3.659∗
		(0.182)	(0.077)	(0.185)	(0.679)	(0.248)
Log pseudo-likelihood	-4.7e+04					
N	44572					

Notes: ∗, † and ‡ indicate significance at 1, 5 and 10 percent levels respectively. Robust standard errors are in parentheses. Time averaged explanatory variables are included - complete results available from authors.

Table 3: Earnings Model Results

	Boundary Parameters
$\widehat{\gamma_1}$	66
$\widehat{\gamma_2}$	100
$\widehat{\gamma_3}$	35
	Earnings Model
$\widehat{\alpha_1}$	0.124$*$
	(0.001)
$\widehat{\alpha_2}$	0.694$*$
	(0.001)
$\widehat{\theta_{12}}$	-19.457$*$
	(5.570)
$\widehat{\theta_{11}}$	-39.485$*$
	(2.578)
$\widehat{\theta_{21}}$	127.776$*$
	(4.397)
$\widehat{\theta_{23}}$	49.692$*$
	(2.619)
$\widehat{\theta_{24}}$	138.727$*$
	(1.653)
Log likelihood	-1.0e+06
N	44572

Notes: Pooled data from BHPS 1991-2008; All income variables are expressed in 2008 prices; Standard errors in parentheses are bootstrap estimates from 200 replications; $*$, † and ‡ indicate significance at 1, 5 and 10 percent levels respectively; Time averaged explanatory variables are included - complete results available from authors.

Table 4: Labor Supply Elasticities (Evaluated at the Sample Mean)

	Single Job (unconstrained)	**Dual Job (unconstrained)**		**Dual Job (underemployed)**	**Dual Job (overemployed)**
	Job 1	Job 1	Job 2	Job 2	Job 2
η_{ii}	0.027	0.172	3.748	2.358	2.574
η_{ij}		-0.161	-3.577	-2.146	-2.424
η_{iI}	-0.056	-0.021	-0.328	-0.434	-0.304
η_{ii}^c	0.431	0.295	4.442	3.150	3.366
η_{ij}^c		-0.117	-1.616	0.000	0.000
ϵ_{iiI}	-0.404	-0.124	-0.694	-0.792	-0.792
ϵ_{ijI}		-0.044	-1.962	-2.146	-2.424

Notes: Based on BHPS 1991-2008.

A Appendix

Table A.1: Labor Supply Elasticities (Average Elasticities across Individuals)

	Single Job (unconstrained)	**Dual Job (unconstrained)**		**Dual Job (underemployed)**	**Dual Job (overemployed)**
	Job 1	Job 1	Job 2	Job 2	Job 2
η_{ii}	0.041	0.238	2.945	12.715	12.788
η_{ij}		-0.222	-3.731	-12.182	-12.273
η_{iI}	-0.078	-0.030	0.651	-1.798	-1.373
η^c_{ii}	0.445	0.361	3.639	13.507	13.580
η^c_{ij}		-0.157	-1.691	0.000	0.000
ϵ_{iiI}	-0.404	-0.124	-0.694	-0.792	-0.792
ϵ_{ijI}		-0.065	-2.041	-12.182	-12.273

Notes: Based on BHPS 1991-2008.

Table A.2: Labor Supply Elasticities (Median Elasticities across Individuals)

	Single Job (unconstrained)	**Dual Job (unconstrained)**		**Dual Job (underemployed)**	**Dual Job (overemployed)**
	Job 1	Job 1	Job 2	Job 2	Job 2
η_{ii}	-0.004	0.104	2.660	1.926	2.234
η_{ij}		-0.093	-2.545	-1.789	-2.253
η_{iI}	-0.023	-0.009	-0.065	-0.129	-0.091
η_{ii}^c	0.400	0.228	3.354	2.718	3.026
η_{ij}^c		-0.070	-1.187	0.000	0.000
ϵ_{iiI}	-0.404	-0.124	-0.694	-0.792	-0.792
ϵ_{ijI}		-0.026	-1.367	-1.789	-2.253

Notes: Based on BHPS 1991-2008.

Table A.3: Labor Supply Elasticities (Evaluated at the Sample Median)

	Single Job (unconstrained)	**Dual Job (unconstrained)**		**Dual Job (underemployed)**	**Dual Job (overemployed)**
	Job 1	Job 1	Job 2	Job 2	Job 2
η_{ii}	-0.005	0.087	7.884	3.998	5.001
η_{ij}		-0.089	-7.944	-4.066	-5.069
η_{iI}	-0.028	-0.009	-0.340	-0.365	-0.321
η_{ii}^{c}	0.399	0.211	8.578	4.790	5.793
η_{ij}^{c}		-0.070	-3.402	0.000	0.000
ϵ_{iiI}	-0.404	-0.124	-0.694	-0.792	-0.792
ϵ_{ijI}		-0.019	-4.541	-4.066	-5.069

Notes: Based on BHPS 1991-2008.

B Technical Appendix

In this appendix we present the derivations underlying the labor supply elasticities reported in the body of the paper as well as the empirical model for the hours specification of labor supply.

B.1 Labor Supply Elasticities

Unconstrained dual job holder

The uncompensated own wage effect for job m is given by

$$\frac{\partial h_m^*}{\partial w_m} = \frac{\alpha_m}{(w_m)^2}\left(\gamma_k w_k + \ I \ - \gamma_3\right) \gtreqless 0,\ m, k = 1, 2\ , m \neq k,$$

and in elasticity terms is

$$\begin{aligned}\eta_{mm} &= \frac{w_m}{h_m^*}\frac{\partial h_m^*}{\partial w_m} \\ &= \frac{\alpha_m}{w_m h_m^*}\left(\gamma_k w_k + \ I \ - \gamma_3\right) \gtreqless 0.\end{aligned}$$

The pure income effect is given by

$$\frac{\partial h_m^*}{\partial I} = \frac{-\alpha_m}{w_m} < 0,$$

and in elasticity terms by

$$\begin{aligned}\eta_{mI} \ &= \frac{I}{h_m}\frac{\partial h_m^*}{\partial I} \\ &= \frac{-\alpha_m I}{w_m h_m^*} < 0.\end{aligned}$$

The Slutsky equation decomposition of own wage effects is specified by

$$\frac{\partial h_m^*}{\partial w_m} = S_{mm} + h_m^* \frac{\partial h_m^*}{\partial I},$$

where S_{mm} is the own (compensated) substitution effect, and

$$h_m^* \frac{\partial h_m^*}{\partial I} = \frac{-\alpha_m h_m^*}{w_m} < 0$$

is the own wage income effect. It follows that the own substitution effect can be obtained residually from the Slutsky equation:

$$\begin{aligned}S_{mm} &= \frac{\partial h_m^*}{\partial w_m} - h_m^* \frac{\partial h_m^*}{\partial I} \\ &= \frac{\alpha_m}{(w_m)^2}\left(\gamma_k w_k + w_m h_m^* + \ I \ - \gamma_3\right) > 0.\end{aligned}$$

It is easy to show that $S_{mm} > 0$ because

$$S_{mm} = \frac{\alpha_m}{(w_m)^2}\left(\gamma_k w_k + w_m h_m^* + \ I \ - \gamma_3\right) > \frac{\alpha_m}{(w_m)^2}\left(y^* - \gamma_3\right) > 0.$$

The compensated own wage (subsitution effect) elasticity is given by

$$\begin{aligned}\eta_{mm}^c &= \frac{w_m}{h_m^*} S_{mm} \\ &= \frac{\alpha_m}{w_m h_m^*}\left(\gamma_k w_k + w_m h_m^* + \ I \ - \gamma_3\right) > 0.\end{aligned}$$

In elasticity terms the Slutsky equation may be expressed as

$$\begin{aligned}\eta_{mm} &= \frac{w_m}{h_m^*}\left(S_{mm} + h_m^* \frac{\partial h_m^*}{\partial I}\right) \\ &= \frac{w_m}{h_m^*} S_{mm} + w_m \frac{\partial h_m^*}{\partial I} \\ &= \eta_{mm}^c + \epsilon_{mmI} \ \gtreqless 0.\end{aligned}$$

where the own wage income effect elasticity (ϵ_{mmI}) is obtained from

$$\begin{aligned}\epsilon_{mmI} \ &= w_m \frac{\partial h_m^*}{\partial I} \\ &= -\alpha_m \ < 0.\end{aligned}$$

The uncompensated cross wage effect on the supply of labor to job m from a change in the wage for job k is calculated according to

$$\frac{\partial h_m^*}{\partial w_k} = \frac{-\alpha_m \gamma_k}{w_m} < 0, \ m \neq k$$

and in elasticity terms by

$$\begin{aligned}\eta_{mk} &= \frac{w_k}{h_m^*} \frac{\partial h_m^*}{\partial w_k} \\ &= \frac{-\alpha_m \gamma_k w_k}{w_m h_m^*} < 0.\end{aligned}$$

The Slutsky equation decomposition of cross wage effects is given by

$$\frac{\partial h_m^*}{\partial w_k} = S_{mk} + h_k^* \frac{\partial h_m^*}{\partial I},$$

where S_{mk} is the income compensated cross-wage effect, and

$$h_k^* \frac{\partial h_m^*}{\partial I} = \frac{-\alpha_m h_k^*}{w_m} < 0$$

is cross wage income effect. It follows that the compensated cross wage effect can be obtained residually from the Slutsky equation:

$$\begin{aligned} S_{mk} &= \frac{\partial h_m^*}{\partial w_k} - h_k^* \frac{\partial h_m^*}{\partial I} \\ &= \frac{\alpha_m}{w_m} \left(h_k^* - \gamma_k \right) < 0. \end{aligned}$$

It is easily shown that the familiar symmetry property holds for compensated cross wage effects by substituting for h_k^* and h_m^* in the following expression:

$$S_{mk} = \frac{\alpha_m}{w_m} \left(h_k^* - \gamma_k \right) = \frac{\alpha_k}{w_k} \left(h_m^* - \gamma_m \right) = S_{km}.$$

The income compensated cross wage elasticities are obtained from the Slutsky equation in elasticity form:

$$\begin{aligned} \eta_{mk} &= \frac{w_k}{h_m^*} \left(S_{mk} + h_k^* \frac{\partial h_m^*}{\partial I} \right) \\ &= \frac{w_k}{h_m^*} S_{mk} + \frac{h_k^*}{h_m^*} \frac{\partial h_m^*}{\partial I} \\ &= \eta_{mk}^c + \epsilon_{mkI} \ , \end{aligned}$$

where η_{mk}^c is the compensated cross subsititution effect elasticity determined by

$$\begin{aligned} \eta_{mk}^c &= \frac{w_k}{h_m^*} S_{mk} \\ &= \frac{\alpha_m w_k}{w_m h_m^*} \left(h_k^* - \gamma_k \right) < 0, \end{aligned}$$

and ϵ_{mkI} is the compensated cross income effect elasticity determined by

$$\begin{aligned} \epsilon_{mkI} &= \frac{h_k^*}{h_m^*} \frac{\partial h_m^*}{\partial I} \\ &= \frac{-\alpha_m w_k h_k^*}{w_m h_m^*} < 0. \end{aligned}$$

Unconstrained unitary job holders

The uncompensated own wage effect for job 1 is determined according to

$$\frac{\partial h_1^*}{\partial w_1} |_{h_2=0} = \left(\frac{\alpha_1}{1-\alpha_2} \right) \left(\frac{1}{w_1} \right)^2 (\ I \ - \gamma_3) \gtreqless 0$$

and in elasticity terms by

$$\begin{aligned}\eta_{11}\,|_{h_2=0} &= \frac{w_1}{h_1^*}\frac{\partial h_1^*}{\partial w_1}\,|_{h_2=0} \\ &= \left(\frac{\alpha_1}{1-\alpha_2}\right)\left(\frac{1}{w_1 h_1^*}\right)(\,I\;-\gamma_3) \gtreqless 0.\end{aligned}$$

The pure income effect for h_1^* is given by

$$\frac{\partial h_1}{\partial I}\,|_{h_2=0} = \left(\frac{-\alpha_1}{1-\alpha_2}\right)\left(\frac{1}{w_1}\right) < 0,$$

and in elasticity terms by

$$\begin{aligned}\eta_{1I}\,|_{h_2=0} &= \frac{I}{h_1^*}\frac{\partial h_1^*}{\partial I}\,|_{h_2=0} \\ &= \left(\frac{-\alpha_1}{1-\alpha_2}\right)\left(\frac{I}{w_1 h_1^*}\right) < 0.\end{aligned}$$

The Slutsky equation decomposition of own wage effects is specified by

$$\frac{\partial h_1^*}{\partial w_1}\,|_{h_2=0} = S_{11}\,|_{h_2=0} + h_1^*\frac{\partial h_1^*}{\partial I}\,|_{h_2=0}$$

where $S_{11}\,|_{h_2=0}$ is the own (compensated) substitution effect, and

$$h_1^*\frac{\partial h_1^*}{\partial I}\,|_{h_2=0} = \left(\frac{-\alpha_1}{1-\alpha_2}\right)\left(\frac{h_1^*}{w_1}\right) < 0$$

is the own wage income effect. It follows that the own substitution effect can be obtained residually from the Slutsky equation:

$$\begin{aligned}S_{11}\,|_{h_2=0} &= \frac{\partial h_1^*}{\partial w_1}\,|_{h_2=0} - h_1^*\frac{\partial h_1^*}{\partial I}\,|_{h_2=0} \\ &= \left(\frac{\alpha_1}{1-\alpha_2}\right)\left(\frac{1}{w_1}\right)^2 (w_1 h_1^* + I - \gamma_3) > 0.\end{aligned}$$

The compensated own wage (subsitution effect) elasticity is given by

$$\begin{aligned}\eta_{11}^c\,|_{h_2=0} &= \frac{w_1}{h_1^*}S_{11}\,|_{h_2=0} \\ &= \left(\frac{\alpha_1}{1-\alpha_2}\right)\left(\frac{1}{w_1 h_1}\right)(w_1 h_1 + I - \gamma_3) > 0.\end{aligned}$$

The Slutsky equation may be expressed in elasticity terms as

$$\begin{aligned}\eta_{11} &= \frac{w_1}{h_1^*}\left(S_{11}\,|_{h_2=0} + h_1^*\frac{\partial h_1^*}{\partial I}\,|_{h_2=0}\right) \\ &= \frac{w_1}{h_1^*}S_{11}\,|_{h_2=0} + w_1\frac{\partial h_1^*}{\partial I}\,|_{h_2=0} \\ &= \eta_{11}^c\,|_{h_2=0} + \epsilon_{11I}\,|_{h_2=0}\ ,\end{aligned}$$

where $\epsilon_{11I}\,|_{h_2=0}$ is the income effect elasticity from the own wage:

$$\begin{aligned}\epsilon_{11I}\,|_{h_2=0} &= w_1\frac{\partial h_1^*}{\partial I}\,|_{h_2=0} \\ &= \frac{-\alpha_1}{1-\alpha_2} < 0.\end{aligned}$$

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Constrained dual job holder

The uncompensated wage and income effects for labor supply to the second job when constrained on the first job are given below.

The uncompensated own wage effect for job 2 is determined according to

$$\frac{\partial h_2^*}{\partial w_2}\,|_{h_1=\dot{h}_1} = \left(\frac{\alpha_2}{1-\alpha_1}\right)\left(\frac{1}{w_2}\right)^2\left(w_1\dot{h}_1 + I - \gamma_3\right) \gtreqless 0$$

and in elasticity terms by

$$\begin{aligned}\eta_{22}\,|_{h_1=\dot{h}_1} &= \frac{w_2}{h_2^*}\frac{\partial h_2^*}{\partial w_2}\,|_{h_1=\dot{h}_1} \\ &= \left(\frac{\alpha_2}{1-\alpha_1}\right)\left(\frac{1}{w_2h_2^*}\right)\left(w_1\dot{h}_1 + I - \gamma_3\right) \gtreqless 0.\end{aligned}$$

The pure income effect for h_2^* is determined according to

$$\frac{\partial h_2^*}{\partial I}\,|_{h_1=\dot{h}_1} = \left(\frac{-\alpha_2}{1-\alpha_1}\right)\left(\frac{1}{w_2}\right) < 0,$$

and in elasticity terms by

$$\begin{aligned}\eta_{2I}\,|_{h_1=\dot{h}_1} &= \frac{I}{h_2}\frac{\partial h_2^*}{\partial I}\,|_{h_1=\dot{h}_1} \\ &= \left(\frac{-\alpha_2}{1-\alpha_1}\right)\left(\frac{I}{w_2h_2^*}\right) < 0.\end{aligned}$$

The Slutsky equation decomposition of own wage effects for job 2 is specified by

$$\frac{\partial h_2^*}{\partial w_2}\,|_{h_1=\dot{h}_1} = S_{22}\,|_{h_1=\dot{h}_1} + h_2^*\frac{\partial h_2}{\partial I}\,|_{h_1=\dot{h}_1}$$

where $S_{22}\left|_{h_1=\dot{h}_1}\right.$ is the own (compensated) substitution effect, and

$$h_2^* \frac{\partial h_2}{\partial I}\left|_{h_1=\dot{h}_1}\right. = \left(\frac{-\alpha_2}{1-\alpha_1}\right)\left(\frac{\dot{h}_1}{w_2}\right) < 0$$

is the own wage income effect. It follows that the own substitution effect can be obtained residually from the Slutsky equation:

$$\begin{aligned} S_{22}\left|_{h_1=\dot{h}_1}\right. &= \frac{\partial h_2^*}{\partial w_2}\left|_{h_1=\dot{h}_1}\right. - h_2^* \frac{\partial h_2^*}{\partial I}\left|_{h_1=\dot{h}_1}\right. \\ &= \left(\frac{\alpha_2}{1-\alpha_1}\right)\left(\frac{1}{w_2}\right)^2 \left(w_1\dot{h}_1 + w_2 h_2^* + I - \gamma_3\right) > 0. \end{aligned}$$

The compensated own wage (subsitution effect) elasticity is given by

$$\begin{aligned} \eta_{22}^c\left|_{h_1=\dot{h}_1}\right. &= \frac{w_2}{h_2^*} S_{22}\left|_{h_1=\dot{h}_1}\right. \\ &= \left(\frac{\alpha_2}{1-\alpha_1}\right)\left(\frac{1}{w_2 h_2^*}\right)\left(w_1\dot{h}_1 + w_2 h_2^* + I - \gamma_3\right) > 0. \end{aligned}$$

In elasticity terms the Slutsky equation may be expressed as

$$\begin{aligned} \eta_{22}\left|_{h_1=\dot{h}_1}\right. &= \frac{w_2}{h_2^*}\left(S_{22}\left|_{h_1=\dot{h}_1}\right. + h_2^* \frac{\partial h_2}{\partial I}\left|_{h_1=\dot{h}_1}\right.\right) \\ &= \frac{w_2}{h_2^*} S_{22}\left|_{h_1=\dot{h}_1}\right. + w_2 \frac{\partial h_2^*}{\partial I}\left|_{h_1=\dot{h}_1}\right. \\ &= \eta_{22}^c\left|_{h_1=\dot{h}_1}\right. + \epsilon_{22I}\left|_{h_1=\dot{h}_1}\right. \end{aligned}$$

where $\epsilon_{22I}\left|_{h_1=\dot{h}_1}\right.$ is the income effect elasticity from the own wage:

$$\begin{aligned} \epsilon_{22I}\left|_{h_1=\dot{h}_1}\right. &= w_2 \frac{\partial h_2^*}{\partial I}\left|_{h_1=\dot{h}_1}\right. \\ &= \frac{-\alpha_2}{1-\alpha_1}. \end{aligned}$$

The uncompensated cross-wage effect of w_1 on h_2 is determined according to

$$\frac{\partial h_2^*}{\partial w_1}\left|_{h_1=\dot{h}_1}\right. = \left(\frac{-\alpha_2}{1-\alpha_1}\right)\left(\frac{\dot{h}_1}{w_2}\right) < 0,$$

or in elasticity terms

$$\begin{aligned} \eta_{21}\left|_{h_1=\dot{h}_1}\right. &= \frac{w_1}{h_2} \frac{\partial h_2^*}{\partial w_1}\left|_{h_1=\dot{h}_1}\right. \\ &= \left(\frac{-\alpha_2}{1-\alpha_1}\right)\left(\frac{w_1\dot{h}_1}{w_2 h_2^*}\right) < 0. \end{aligned}$$

The Slutsky equation decomposition of cross wage effects is given by

$$\frac{\partial h_2^*}{\partial w_1} \Big|_{h_1=\dot{h}_1} = S_{21} \Big|_{h_1=\dot{h}_1} + \dot{h}_1 \frac{\partial h_2^*}{\partial I} \Big|_{h_1=\dot{h}_1}$$

where $S_{21} \Big|_{h_1=\dot{h}_1}$ is the income compensated cross-wage effect, and

$$\dot{h}_1 \frac{\partial h_2^*}{\partial I} \Big|_{h_1=\dot{h}_1} = \left(\frac{-\alpha_2}{1-\alpha_1}\right)\left(\frac{\dot{h}_1}{w_2}\right) < 0$$

is the cross wage income effect. Because the hours on job 1 are constrained, the job 1 wage has only an income effect on labor supply to job 2. Hence the compensated cross substitution effect is zero. This is evident from

$$\frac{\partial h_2^*}{\partial w_1} \Big|_{h_1=\dot{h}_1} = \left(\frac{-\alpha_2}{1-\alpha_1}\right)\left(\frac{\dot{h}_1}{w_2}\right) = \dot{h}_1 \frac{\partial h_2^*}{\partial I} \Big|_{h_1=\dot{h}_1}$$

which implies that $S_{21} \Big|_{h_1=\dot{h}_1} = 0$.

B.2 Empirical Model for Hours

In the case of the multinomial logit model with the hours specification, we define

$$x_{it} = \left(\frac{1}{w_{1it}}, \frac{I_{it}}{w_{1it}}, \text{Age}_{it}, \text{Educ}_{it}, \text{MS}_{it}, \text{DP}_{it}\right)$$

and

$$\bar{x}_i = \left(1, \overline{\left(\frac{1}{w_1}\right)}_i, \overline{\left(\frac{I}{w_1}\right)}_i, \overline{\text{Age}}_i, \overline{\text{Educ}}_i, \overline{\text{MS}}_i, \overline{\text{DP}}_i\right).$$

Unconstrained dual job holders

$$h_{1it} - \tilde{\gamma}_1 = \alpha_1 q_{11it}^h + \theta_{11}^h \hat{\lambda}_{1it}^h + \bar{Z}_{11i}^h \pi_{11} + u_{11it} \tag{16}$$

$$h_{2it} - \tilde{\gamma}_2 = \alpha_2 q_{21it}^h + \theta_{21}^h \hat{\lambda}_{1it}^h + \bar{Z}_{21}^h \pi_{21} + u_{21it}^h. \tag{17}$$

where

$$q_{m1it}^h = \left(\frac{\tilde{\gamma}_3 - \tilde{\gamma}_1 w_{1it} - \tilde{\gamma}_2 w_{2it} - I_{it}}{w_{mit}}\right),$$

$$\hat{\lambda}_{11it}^h = \hat{\lambda}_{21it}^h = \hat{\lambda}_{1it}^h,$$

$$\bar{Z}_{11i}^h = \left(1, \overline{\left(\frac{1}{w_1}\right)}_i, \overline{\left(\frac{w_2}{w_1}\right)}_i, \overline{\left(\frac{I}{w_1}\right)}_i, \overline{\text{Age}}_i, \overline{\text{Educ}}_i, \overline{\text{MS}}_i, \overline{\text{DP}}_i\right),$$

$$\bar{Z}_{21}^h = \left(1, \overline{\left(\frac{1}{w_2}\right)}_i, \overline{\left(\frac{w_1}{w_2}\right)}_i, \overline{\left(\frac{I}{w_2}\right)}_i, \overline{\text{Age}}_i, \overline{\text{Educ}}_i, \overline{\text{MS}}_i, \overline{\text{DP}}_i\right),$$

and π^h_{11} and π^h_{21} are the corresponding parameter vectors.

Unconstrained single job holders

$$h_{1it} - \tilde{\gamma}_1 = \left(\frac{\alpha_1}{1-\alpha_2}\right) q^h_{12it} + \theta^h_{12}\hat{\lambda}^h_{2it} + \bar{Z}^h_{12i}\pi^h_{12} + u^h_{12it} \tag{18}$$

where

$$q^h_{12it} = \left(\frac{\tilde{\gamma}_3 - \tilde{\gamma}_1 w_{1it} - I_{it}}{w_{1it}}\right),$$

$$\bar{Z}^h_{12i} = \left(1, \overline{\left(\frac{1}{w_1}\right)_i}, \overline{\left(\frac{I}{w_1}\right)_i}, \overline{\text{Age}}_i, \overline{\text{Educ}}_i, \overline{\text{MS}}_i, \overline{\text{DP}}_i\right),$$

and π^h_{12} is the corresponding parameter vector.

Constrained dual job holders

Labor supply for those desiring fewer hours and more hours:

$$h_{2it} - \tilde{\gamma}_2 = \left(\frac{\alpha_2}{1-\alpha_1}\right) q^h_{2it} + \theta^h_{23}\hat{\lambda}^h_{23it} + \bar{Z}^h_{2i}\pi^h_{23} + u_{23it} \tag{19}$$

$$h_{2it} - \tilde{\gamma}_2 = \left(\frac{\alpha_2}{1-\alpha_1}\right) q^h_{2it} + \theta_{24}\hat{\lambda}^h_{24it} + \bar{Z}^h_{2i}\pi^h_{24} + u^h_{24it}, \tag{20}$$

where $\dot{h}_{1it}$ is the constrained hours on job 1,

$$\begin{aligned} q^h_{23it} = q^h_{24it} &= q^h_{2it} \\ &= \left(\frac{\tilde{\gamma}_3 - \tilde{\gamma}_2 w_{2it} - w_{1i}\dot{h}_{1it} - I_{it}}{w_{2it}}\right), \end{aligned}$$

$$\begin{aligned} \bar{Z}^h_{23i} = \bar{Z}^h_{24i} &= \bar{Z}^h_{2i} \\ &= \left(1, \overline{\left(\frac{1}{w_2}\right)_i}, \overline{\left(\frac{w_1\dot{h}_1}{w_2}\right)_i}, \overline{\left(\frac{I}{w_2}\right)_i}, \overline{\text{Age}}_i, \overline{\text{Educ}}_i, \overline{\text{MS}}_i, \overline{\text{DP}}_i\right), \end{aligned}$$

and π^h_{23} and π^h_{24} are the corresponding parameter vectors.

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